3D multi-scales X-FEM dedicated to tribological fatigue
Applications to fretting and rolling contact

Anthony Gravouil, M.C. Baietto, B. Trollé, E. Pierres, R. Ribeaucourt

LaMCoS, INSA-LYON, Université de LYON, CNRS UMR 5259
Institut Universitaire de France
Introduction

Two scales X-FEM

Fretting fatigue

Rolling contact fatigue

Conclusions

Fretting Fatigue

- Fretting damage is due to microslip at the interface of contacts experiencing oscillatory loads or vibrations (like riveted joints, blade to disk fixings in jet engines, rolling bearings)

- Degradation: wear and or crack at the two-body interface.

- Damage depending on contact conditions:

  - Fretting Fatigue (partial slip)
    - Crack propagation due to contact fatigue
    - [Vincent et al., ASTM STP 1159, 1992]
    - [Fouvry et al., Wear 1996]

  - Fretting Wear (gross slip)
    - Surface wear: modification of component dimensions or link locking by accumulation of debris
**Introduction**

- Two scales X-FEM
- Fretting fatigue
- Rolling contact fatigue
- Conclusions

---

**Modelling and Numerical simulation of the corresponding fracture problem:**

- **3D structure,**
- **Contact loading : cyclic, multi-axial, non proportional loading,**
- **3D complex crack shapes,**
- **Multiscale nonlinear phenomenon,**
- **contact with friction between crack faces,**
- **Fatigue crack growth law specific to fretting.**
Rolling Contact Fatigue Crack in Rails

- **Rolling contact fatigue**
  - **Rolling**
  - **Sliding**
  - **High contact pressure ~ 1 GPa**
  - **High contact gradient**

- **Rolling contact fatigue (RCF) cracks in rails**
  - **Alignment**
  - **Running sub surface**
  - **Possible multiple cracks**

[IN 0285, SNCF]
Orientation criteria and propagation laws

- **Orientation criteria for multi-axial loading:**
  - **Maximum tangential stress** [Erdogan et al., JBE, 1963]
  - **Maximum shear stress** [Otsuka et al., EFM, 1975]
  - **Amestoy criterion** [Amestoy et al., CRAS, 1979]

- **Propagation laws**
  - **Uni-axial testing machine:**
    - Multi-axial
    - Proportional
  - **Bi-axial testing machine:**
    - Multi-axial
    - Non proportional
  - **Twin discs testing machine:**
    - Multi-axial
    - Non proportional
    - Contact loading

\[
\frac{da}{dN} = \Delta K_{eq}^{2} = \Delta K_{tt}^{2}
\]

[Tanaka et al., EFM, 1974] [Bold et al., Wear, 1991] [Wong et al., Wear, 1996]
Outline

1. A two scales X-FEM for contact

2. Application to Fretting fatigue

3. Application to rolling contact fatigue

4. Conclusions & prospects
A two scales X-FEM for contact
1. eXtended Finite Element Method + level sets

- **Local partition of unity (two scale strategy):**
  \[ u_h = u_{hFEM} + u_{h} \big|_{\text{enrichment}} \]

- **Discontinuous and asymptotic enrichment of the displacement field**
  \[ u_h(x,t) = \left( \sum_{i \in \mathbb{N}} u_i(t) \varphi_i(x) \right) + \left( \sum_{j \in \mathbb{N}_F} a_j(t) \varphi_j(x) H(x) + \sum_{i \in \mathbb{N}_F} \sum_{j} b'_{ij}(t) \varphi_j(x) F_j(\Phi, \Psi) \right) \]
  \[ F_j = \left\{ \sqrt{r} \sin\left(\frac{\theta}{2}\right), \sqrt{r} \cos\left(\frac{\theta}{2}\right), \sqrt{r} \sin\left(\frac{\theta}{2}\right) \sin(\theta), \sqrt{r} \cos\left(\frac{\theta}{2}\right) \sin(\theta) \right\} \]

- **Crack shape modeling by two level sets:**
  \[- \Phi(x,t) = 0 \quad \Psi(x,t) \leq 0 \quad \text{crack} \]
  \[- \Phi(x,t) = 0 \quad \Psi(x,t) = 0 \quad \text{Crack front} \]

- **Level set update**
  \[ \frac{\partial}{\partial t} \Phi + V_\phi \| \nabla \Phi \| = 0 \]

- **Local orthogonality**
  \[ \nabla \Phi \cdot \nabla \Psi = 0 \]

[Gravouil A., Moës N., Belytschko T., IJNME, 2002] [Sethian 1997]
1. eXtended Finite Element Method + level sets

- Coupling with 3D imaging (X-ray microtomography) and Digital Volume Correlation:

[Rannou et al. CMAME 2010]

[E. Ferrié et al, 2006]
### Modelling of the interfacial Frictional contact with X-FEM

Three mainly formulations of the contact problem with X-FEM:

- **Primal formulation:**
  - Interface refinement
  - Stability

- **Dual formulation:**
  - Coupled scales
  - Conditionally stable

- **Mixed formulation:**
  - Uncoupled scales
  - Conditionally stable
Two scales strategy (Structure / crack)

Global problem \((u, \sigma)\)
- Scale of the structure
- Equilibrium and constitutive law in the bulk (possibly nonlinear)

Local problem \((w, t)\)
- Scale of the crack
- Constitutive law at the interface (unilateral contact, contact with friction)
1 Two scale strategy: three field weak formulation

- Principle of virtual works:
  \[ P_{\text{int}} + P_{\text{ext}} + P_{\text{crack}} + P_{\text{coupling}} = 0 \]
  \[ \forall u^* \in U_0^*, \forall w^* \in W^*, \forall \lambda^* \in \Lambda^*, \forall t \in [0; T] \]

- Three field weak formulation of the fracture problem with frictional contact between the crack faces:
  \[
  0 = -\int_\Omega \sigma(t) : \epsilon(u^*)d\Omega + \int_{\Gamma_t} f_t(t) \cdot u^* dS + \int_{\Gamma_C} \lambda(t) \cdot u^* dS \\
  + \int_{\Gamma_C} (t(t) - \lambda(t)) \cdot w^* dS \\
  + \int_{\Gamma_C} (u(t) - w(t)) \cdot \lambda^* dS
  \]
  \[ \forall u^* \in U_0^*, \forall w^* \in W^*, \forall \lambda^* \in \Lambda^*, \forall t \in [0; T] \]

  - Constitutive law in volume \((u,s)\) (possibly non linear)
  - Frictional contact law at the interface \((w,t)\)

- Allows an intrinsic description of the crack interface:
  - with its own primal and dual variables \((w,t)\)
  - with its own (possibly refined) discretization

Introduction

Two scales X-FEM

Fretting fatigue

Rolling contact fatigue

Conclusions
1. Non linear iterative solver (LATIN method)

- Iterative solver for the solution of the frictional contact problem: \( s = (u, \omega, t) \)
- Divide the equations into two subsets:
  - Global linear equations (G) (3 field weak formulation)
  - Local possibly non linear equations (L) (frictional contact equations)
- Find an approximate solution according to an iterative process in 2 stages

Iterative strategy:

Corresponding search directions:

\[
\begin{align*}
  t_{i+1} - t_i + \frac{1}{2} & = K_0 (\omega_{i+1} - \omega_i) \\
  t_{i+1} - t_i + \frac{1}{2} & = -K_0 (\omega_{i+1} - \omega_{i+\frac{1}{2}}) \\
  K_0 & = k_0 I_d
\end{align*}
\]


1. **Non linear iterative solver (LATIN method)**

   **Local stage (frictional contact equations)**

- **Notations for the local interface fields:**

  \[
  [w] = \omega^- - \omega^+ \\
  \Delta w = \omega^n - \omega^{n-1} \\
  \Delta [w_T] = \Delta \omega_T^{-n} - \Delta \omega_T^{+n} = (\omega_T^{-n} - \omega_T^{+n}) - (\omega_T^{-(n-1)} - \omega_T^{+(n-1)})
  \]

- **Unilateral contact at crack interface (w, t)**

  \[
  [w_N] := \omega_N^- - \omega_N^+ \geq 0 \\
  F_N := t_N^+ = -t_N^- \leq 0 \\
  F_T := t_T^+ = -t_T^- \\
  [w_N] . F_N = 0
  \]

- **Frictional conditions at crack interface (w, t)**

  \[
  \| F_T \| < \mu_C | F_N | \Rightarrow \Delta [w_T] = 0 \\
  \| F_T \| = \mu_C | F_N | \Rightarrow \exists \lambda \geq 0, \Delta [w_T] = \lambda F_T
  \]
1. **Stabilization of the three field weak formulation**

- The X-FEM three field weak formulation can be unstable whatever the non-linear solver.

\[
\begin{bmatrix}
K & 0 & -K_{u\lambda} \\
0 & K_{ww} & -K_{w\lambda} \\
-K_{u\lambda}^T & K_{w\lambda} & 0 \\
\end{bmatrix}
\begin{bmatrix}
U_{i+1} \\
W_{i+1} \\
\Lambda_{i+1} \\
\end{bmatrix}
= 
\begin{bmatrix}
F \\
K_{w\lambda} \cdot T_{i+\frac{1}{2}} + K_{ww} \cdot W_{i+\frac{1}{2}} \\
0 \\
\end{bmatrix}
\]

- Introduction of a stabilization term on the local-global coupling condition in order to satisfy the LBB condition:

\[
\begin{bmatrix}
K & 0 & -K_{u\lambda} \\
0 & K_{ww} & -K_{w\lambda} \\
-K_{u\lambda}^T & K_{w\lambda} & K_{\lambda\lambda} \\
\end{bmatrix}
\begin{bmatrix}
U_{i+1} \\
W_{i+1} \\
\Lambda_{i+1} \\
\end{bmatrix}
= 
\begin{bmatrix}
F \\
K_{w\lambda} \cdot T_{i+\frac{1}{2}} + K_{ww} \cdot W_{i+\frac{1}{2}} \\
K_{\lambda\lambda} \cdot \Lambda_{i} \\
\end{bmatrix}
\]

- The exact solution is obtained at convergence.

- **Stability condition of Ladyzhenskaya-Babuška-Brezzi (LBB):**

\[
\inf_{Z \in \mathcal{Z}} \sup_{Y \in \mathcal{Y}} \frac{Y^T B^T Z}{\|Y\|_Y \cdot \|Z\|_Z} \geq \beta > 0 \\
\text{with} \quad \begin{cases} 
\|Y\|_Y \leq \frac{1}{\alpha M_a} \frac{4 M_b}{\alpha + \beta^2} \cdot \|d\| \\
\|Z\|_Z \leq \frac{4 M_a^{1/2} M_b}{2 M_a^{1/2} \alpha + \alpha^{1/2} \beta M_b} \cdot \|F\| + \frac{4 M_a}{M_a \alpha + \beta^2} \cdot \|d\| 
\end{cases}
\]

Introduction
Two scales X-FEM
Fretting fatigue
Rolling contact fatigue
Conclusions
Two scales X-FEM discretization of the crack interface

- New approach: Independent discretization of the interface
- Discretization of the interface independent of the underlying X-FEM mesh
- Three field weak formulation: Non matching discretizations authorized
- Interface elements divided according to size and shape.

- Uniform distribution of Gauss points along the interface.
  Refinement adapted to the local frictional contact conditions.

[Pierres E., Baietto M.C., Gavouil A., CMAME 2009]
**Example of crack propagation: update of the level sets and definition of new interface elements + calculation of 3D SIFs based on 3D path independent integrals**

\[
I_{\text{R,aux}}^R = \int_D A_{1,j} q_1 \, dV + \int_D A_{1,j} q_{1,j} \, dV + \int_{\Gamma^+ \cup \Gamma^-} t_{2}^{\text{R,aux}} u_{1,1} q_1 \, dS
\]

\[
A_{1,j} = \left( W_{l}^{\text{R,aux}} \delta_{1,j} - \sigma_{1,j}^{\text{R,aux}} u_{1,1} - \sigma_{1,j}^{\text{aux}} u_{1,1} \right)
\]

[Pierres E., Baietto M.C., Gavouil A., CMAME 2009]
1. Efficiency and robustness of the two-scales X-FEM model for frictional cracks

- **Numerical stability of the model**
- **Optimized nonlinear convergence**

- **Accuracy and CPU saving**
- **Efficiency of the Global-local X-FEM**

Introduction
Two scales X-FEM
Fretting fatigue
Rolling contact fatigue
Conclusions
Extension to multigrid XFEM strategies

1 Introduction

Two scales X-FEM

Fretting fatigue

Rolling contact fatigue

Conclusions
Application to fretting fatigue
2 Numerical modeling of experimental fretting fatigue tests.

Numerical simulation with the Global – local X-FEM

Goal: Account for 3D complex crack geometries, local fretting loading, frictional contact conditions, multi-scale effects

Constant normal pressure $P = 50$ MPa and Cyclic tangential pressure $Q_{\text{max}} = 59$ MPa on a circular area on the surface:
Multi–model strategy for the prediction of fretting crack life time

Controled fretting experiments
cylinder/plane or sphere/plane

Data loads are saved
Fretting loop: partial sliding
Calculation of the local friction coefficient $\mu(t)$

Solving of the two-body contact pb:
Calculation of normal and tangential stress fields
($p$ and $q(t)$)

Crack initiation locations and angles:
Dang Van multi-axial fatigue criterion

Global - Local X-FEM
Stress Intensity Factors
2D and 3D crack propagation

[Fouvy et al. Wear 1996]

[DangVan, ASTM STP 1993]
2D 3D fretting fatigue experiments: sphere / plane contact

Introduction
Two scales X-FEM
Fretting fatigue
Rolling contact fatigue
Conclusions

Sphere / plane experiment

Experimental fretting crack

Transversal cut

3D crack shape reconstruction
Global – Local X-FEM Simulation of a 3D fretting fatigue test

- **Specimen:**
  - $25\text{mm} \times 16\text{mm} \times 4\text{mm}$
  - *Steel*: $E = 210 \text{ GPa}$; $\nu = 0.3$
  - *Mesh*: $46266$ tetraedra
  - *local refinement close to the area of interest (contact zone)*

- **Crack:**
  - *level sets*
  - *Discretization*: $2574$ interface elements
  - *boundary crack length*: $\sim 600 \mu\text{m}$
  - *in the bulk*: $\sim 100 \mu\text{m}$

- **Loading:**
  - *obtained from the two-body contact calculation*
  - *time discretization for 1 cycle*: $25$ time steps
Global – Local X-FEM Simulation of a 3D fretting fatigue test

Fretting cycle

Global displacement field

contact load

Local sliding

Local crack opening
2. **X-FEM model: Crack Propagation and Orientation criteria**

- **Experimental studies**
  - Experimental data derived from [Pierres, PhD, 2010]
  - Experimental crack path
  - Mixed mode propagation law dedicated to fretting application

- **Numerical simulations**
  - \( \max \Delta k^*_1 (\theta,t) \) → based on local maxima in space and time, not suitable for non-proportional loading
  - \( \max k^*_1 (\theta,t) \) → based on amplitude over the cycle
  - \( \max \frac{da}{dN} (\theta,t) \) → based on amplitude over the cycle

[Baietto et al, IJF, 2013]
Application to rolling contact fatigue
Available 3D Numerical Tools (IdR2)

- **Vocolin**
  - Rail vehicle dynamics

- **Starail**
  - Finite element modeling

- **Mxfat**
  - Fatigue analysis: Dang Van Criterion

- **Industrial requirement: CAST3M software**

- **Vehicle Track Wheel profile Rail profile**

- **Contact load on the rail**

- **Residual stresses**

- **Initial macro crack**

- **Crack propagation**

- **Train traffic**

- **Crack initiation**

- **Propagation**
3 Quantitative prediction of the rolling contact fatigue crack behavior in rails

- 3D Multi-scale problem

- RCF: a multi-axial non-proportional loading

- 3D crack propagation simulations, not only fracture analysis
**RCF application: Fatigue Cycle Simulation**

- **Hertzian travelling load defined by user or derived from dedicated software**
  
  ![Extraction displacement](image)

  
  
- **Travelling load and crack scale: multi-scale parametric mesh**
  
  - Semi-infinite domain
  - Contact gradient
  - K-dominance zone

- **Quasi-static simulation**

- **One cycle = one wheel travelling over the crack**
3 RCF application: comparison with existing results

- Comparison with a semi-analytic model
  [Dubourg et al, JT, 2002]

- Wheel-rail contact pressure: cylinder-plane contact
  - Fully sliding Herztian loading
    - $P_{\text{max}} = 845 \text{ Mpa}$
    - $2a = 13.5 \text{ mm}$
    - $\mu_{\text{wheel/rail}} = 0.4$

- SIFs in the initial configuration

\[
K_{II} (\text{Mpa})
\]
\[
K_{I} (\text{Mpa})
\]
3 RCF application: 3D example

- **Fully sliding Hertzian loading**
  - Vertical semi-circular crack
  - $\mu_c = 0$

- **Global displacement field**

- **Traction local field**

- **Structure and crack meshes**

- **Stress intensity factors**

- **Parameters**
  - $P_{\text{max}} = 1348$ MPa
  - $2a = 6.75$ mm
  - $2b = 4.7$
  - $\mu_{\text{wheel/rail}} = 0.3$
3 RCF application: 2D and 3D residual plastic stresses

- Plastic strain accumulation due to the repeated solicitations and initial plasticity from the manufacturing process

- Most of the time, elastic shakedown is predicted

- Only elastic shakedown is considered

- Complex plastic fields with high gradients

- High compressive stresses just under the running surface (~5 mm thick)

- « C-shape » in the rail web, characteristic of the manufacturing process
3 RCF application: plastic stresses

- Plastic stress projection

   - Vocabulary:
     - Vocolin
     - Rail vehicle dynamics
     - Starail
     - Finite element modeling

   - Contact load on the rail

   - Asymptotic stresses

   - Different meshes → Projection

   - Solution of the cracked structure problem

- Plastic stresses are considered as an initial non-uniform permanent state

  [Nguyen, IJNME, 1977]  [Dang Van et al, JMPS, 1993]
3 RCF application: plastic stresses, 2D reference case

- **Plastic stresses effect on the reference case**
  [Dubourg et al., JT, 2002]

- **Plastic stress field**

  ![Graph showing compressive and tensile stresses](image)

- **Modification of the solicitations:**
  - Crack always closed ($K_I = 0$)
  - Modification of $K_{II}$ values, plastic stresses influence the sliding between the crack faces

- **Residual plastic stresses have to be to taken into account**

- **Fully sliding Herzian loading**
  - $P_{max} = 845\, MPa$
  - $2a = 13.5\, mm$
  - $\mu_{\text{wheel/rail}} = 0.4$

- **With plastic stresses**
  - $K_{II} (MPa)$
  - $K_I (MPa)$
3 Crack propagation under RCF: tangential loading

- Influence of the wheel-rail contact friction coefficient
  - $\mu_{\text{wheel-rail}} = 0.025 ; 0.1 ; 0.4$

- $l = 6 \text{ mm}$
- $\theta = 15^\circ$
- $\mu_c = 0.5$

A high tangential loading enhances mode I and mode II
$\Delta K_{\text{II}}$ increases faster than $\Delta K_{\text{I}}$ (geometry effect)

[Trollé et al, EFM, 2014]
Crack propagation under RCF: tangential loading

- Influence of the wheel-rail contact friction coefficient
  - $\mu_{\text{wheel-rail}} = 0.025; 0.1; 0.4$
  - $l = 6\,\text{mm}$
  - $\theta = 15^\circ$
  - $\mu_c = 0.5$

Cracks propagate towards the running surface

A higher mode II enhances earlier crack branching

Tangential loading direction is responsible for the global crack propagation path

[Trollé et al., EFM2014]
3 Crack propagation under RCF: tangential loading

- **Influence of the initial crack angle**
  - \( \theta = 15°, 30°, 45°, 60°, 75° \)

- **In tangential loading**
  - \( \mu_{\text{wheel-rail}} = 0.1 \)
  - \( \mu_{\text{c}} = 0.5 \)

- **For initial angle \( \geq 30° \), cracks always propagate towards the rail web with an angle between 10° and 30° with the vertical**

- **Relevant with the US detection devices, crack oriented with an angle of 22°**
3 Crack propagation under RCF

- Influence of the plastic stresses (with and without plastic stress)

- Modification of crack growth path and decreasing of $K_{II}$ avoiding crack branching
  -> longer coplanar propagation

- Crack twice slower with plastic stresses

- Actual plastic stresses have to be taken into account
3 Crack propagation under RCF

Crack network: study of a « squat configuration »

Traffic direction

\[\text{[Cannon1996]}\]

Traffic direction

\[\text{[Steenbergen2013]}\]

Traffic direction

\[\text{[Grassie2012]}\]
3 Crack propagation under RCF

- Crack network: study of a « squat configuration »

\[\text{Predicted numerical paths in accordance with the experimental observations}\]
3 Crack network

- Crack network effect
- Plastic stresses have to be taken into account

**Crack network without plastic stresses**

- Free
- With plastic stresses

**Crack network with plastic stresses**

Traffic direction

\[ l = 6 \text{ mm} \]
\[ \theta = 15^\circ \]
\[ \mu_c = 0.5 \]
3 Crack network

- Prediction of 3D fatigue crack propagation of squat and headcheck configurations with rolling contact fatigue
Conclusions & prospects
Conclusions and prospects

- **Fully integrated coupled tool for Fretting Fatigue and Rolling Contact Fatigue**
  
  - [Bogdanski et al, Wear, 2005]
  - [Bogdanski et al, Wear, 2008]
  - [Fletcher et al, EFM, 2008]
  - [Balcombe et al, Wear, 2011]
  - [Farjoo et al, EFM, 2012]

- **Fluid entrapment and tip pressurization**

- **3D propagation law and 3D orientation criteria**

- **Crack initiation and crack propagation in the same model**
  
  - [Rannou et al, CMAME, 2010]
  - [Réthoré et al, PIUTAM, 2012]

- **Influence of the microstructure on the crack initiation**
  
  - [Bernard et al, CMAME, 2012]
  - [Noyel et al, 2016]
  - [Fouvry, 2016]

- **Reduced order modeling for massive parametric studies and off-line post-processing on reduced basis**
  
  - [Galland et al, CMAME, 2011]
Merci pour votre attention !